Calculator Free
Differentiation Techniques

Time: 45 minutes
Total Marks: 45
Your Score: / 45

Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] CF

Differentiate each of the following functions with respect to *x*. Do not simplify your answers.

(a) $y = e^{-3x}$

(b) $g(x) = -\cos\left(\frac{x}{2}\right)$

(c) $f(x) = x^2 e^{2x-1}$

(d) $y = \frac{\sin x}{5x-1}$

(e) $h(x)=\sqrt{x^4-2x}$

(f) $y=\sin^2(4x)$

(g) $y=2f(3x-1)$

Question Two: **[4 marks]** **CF**

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

Question Three: **[4 marks]** **CF**

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of *y* with respect to *x*, in terms of *x* only.
Simplify your answer.

Question Four: [5 marks] CF

Given that $y = e^{x^2 - 1}$, show that $\frac{d^2y}{dx^2} \times y^{-1} - 2 = 4x^2$

Question Five: [2 marks] CF

Given $f\,'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

Question Six: [5 marks] CF

By using first principles and the limits $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, establish that $\frac{d}{dx}\sin x = \cos x$.

Remember that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

Question Seven: [3, 4 = 7 marks] CF

(a) Calculate the gradient of the curve $y = \dfrac{e^{-2x}}{5x}$ at $x = -1$.

(b) Determine the equation of the tangent to the curve $f(x) = -\cos(4x)$ at $x = \dfrac{\pi}{6}$.

SOLUTIONS
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Question One: [1, 2, 3, 3, 3, 3, 3 = 18 marks] CF

Differentiate each of the following functions with respect to x. Do not simplify your answers.

(a) $y = e^{-3x}$

$$\frac{dy}{dx} = -3e^{-3x} \checkmark$$

(b) $g(x) = -\cos\left(\frac{x}{2}\right)$

$$g'(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right) \checkmark \checkmark$$

(c) $f(x) = x^2 e^{2x-1}$

$$f'(x) = 2x(e^{2x-1}) + 2x^2 e^{2x-1} \checkmark \checkmark \checkmark$$

(d) $y = \frac{\sin x}{5x-1}$

$$\frac{dy}{dx} = \frac{(5x-1)(\cos x) - (5\sin x)}{(5x-1)^2} \checkmark \checkmark \checkmark$$

(e) $h(x)=\sqrt{x^4-2x}$

$$h'(x)=\frac{1}{2}(x^4-2x)^{\frac{-1}{2}}(4x^3-2)$$

(f) $y=\sin^2(4x)$

$$y=(\sin(4x))^2$$

$$\frac{dy}{dx}=2(\sin(4x))(4\cos(4x))$$

(g) $y=2f(3x-1)$

$$\frac{dy}{dx}=2f'(3x-1)(3)$$

Question Two: [4 marks] CF

Show, using the quotient rule, that $\frac{d}{dx}\tan(x) = 1 + \tan^2 x$.

$$y = \tan x = \frac{\sin x}{\cos x}$$
$$\frac{dy}{dx} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2 x}$$
$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$
$$\frac{dy}{dx} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$
$$\frac{dy}{dx} = 1 + \tan^2 x$$

Question Three: [4 marks] CF

A curve is defined parametrically as $x = 4t$ and $y = t^3 - 1$.

Determine an expression for the rate of change of *y* with respect to *x*, in terms of *x* only. Simplify your answer.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$\frac{dy}{dx} = 3t^2 \times \frac{1}{4}$$
$$\frac{dy}{dx} = \frac{3t^2}{4}$$
$$t = \frac{x}{4}$$
$$\therefore \frac{dy}{dx} = \frac{3\left(\frac{x}{4}\right)^2}{4}$$
$$= \frac{3x^2}{16 \times 4}$$
$$= \frac{3x^2}{64}$$

Question Four: [5 marks] CF

Given that $y = e^{x^2-1}$, show that $\frac{d^2y}{dx^2} \times y^{-1} - 2 = 4x^2$

$$\frac{dy}{dx} = 2xe^{x^2-1}$$ ✓

$$\frac{d^2y}{dx^2} = 2e^{x^2-1} + 4x^2e^{x^2-1}$$ ✓

$$= 2e^{x^2-1}(1+2x^2)$$ ✓

$$\frac{d^2y}{dx^2} \times y^{-1} - 2$$ ✓

$$= 2e^{x^2-1}(1+2x^2) \times \frac{1}{e^{x^2-1}} - 2$$

$$= 2 + 4x^2 - 2$$ ✓

$$= 4x^2$$

Question Five: [2 marks] CF

Given $f'(g(x)) = e^{0.5x}\cos(2e^{0.5x})$ and $g(x) = e^{0.5x}$, determine $f(x)$.

$$f(x) = \sin 2x$$

✓✓

Question Six: [5 marks] **CF**

By using first principles and the limits $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$, establish that $\frac{d}{dx}\sin x = \cos x$.

Remember that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$ ✓

$$= \lim_{h \to 0} \frac{\sin x\cosh + \cos x\sinh - \sinx}{h}$$ ✓

$$= \lim_{h \to 0} \frac{\sin x(\cosh - 1) + \cos x\sinh}{h}$$ ✓

$$= \lim_{h \to 0} \frac{\sin x(\cosh - 1)}{h} + \lim_{h \to 0} \frac{\cos x\sinh}{h}$$ ✓

$$= 0 + \cos x$$ ✓

$$= \cos x$$

Question Seven: [3, 4 = 7 marks] CF

(a) Calculate the gradient of the curve $y=\dfrac{e^{-2x}}{5x}$ at $x=-1$.

$$\frac{dy}{dx}=\frac{5x(-2e^{-2x})-5e^{-2x}}{25x^2} ✓✓$$

$$\frac{dy}{dx}=\frac{-5(-2e^{2})-5e^{2}}{25}=\frac{e^2}{5} ✓$$

(b) Determine the equation of the tangent to the curve $f(x)=-\cos(4x)$ at $x=\dfrac{\pi}{6}$.

$$f'(x)=4\sin(4x) ✓$$

$$f'\left(\frac{\pi}{6}\right)=4\sin\frac{2\pi}{3}=2\sqrt{3} ✓$$

$$f\left(\frac{\pi}{6}\right)=\frac{1}{2} ✓$$

$$y=2\sqrt{3}x+c$$

$$\frac{1}{2}=2\sqrt{3}\left(\frac{\pi}{6}\right)+c$$

$$c=\frac{3-2\pi\sqrt{3}}{6}$$

$$\therefore y=2\sqrt{3}x+\frac{3-2\pi\sqrt{3}}{6} ✓$$